

Lectures on Supersymmetry (II)

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Minimal Supersymmetric Standard Model

SM particle	SUSY partner	G_{SM}
$(S = 1/2)$	$(S = 0)$	
$Q = (t, b)_L$	$(\tilde{t}, \tilde{b})_L$	$(3, 2, 1/6)$
$L = (\nu, l)_L$	$(\tilde{\nu}, \tilde{l})_L$	$(1, 2, -1/2)$
$U = (t^C)_L$	\tilde{t}_R^*	$(\bar{3}, 1, -2/3)$
$D = (b^C)_L$	\tilde{b}_R^*	$(\bar{3}, 1, 1/3)$
$E = (l^C)_L$	\tilde{l}_R^*	$(1, 1, 1)$
$(S = 1)$	$(S = 1/2)$	
B_μ	\tilde{B}	$(1, 1, 0)$
W_μ	\tilde{W}	$(1, 3, 0)$
g_μ	\tilde{g}	$(8, 1, 0)$

The Higgs problem

- Problem: What to do with the Higgs field ?
- In the Standard Model masses for the up and down (and lepton) fields are obtained with Yukawa couplings involving H and H^\dagger respectively.
- Impossible to recover this from the Yukawas derived from $P[\Phi]$, since no dependence on $\bar{\Phi}$ is admitted.
- Another problem: In the SM all anomalies cancel,

$$\begin{aligned} \sum_{quarks} Y_i &= 0; & \sum_{left} Y_i &= 0; \\ \sum_i Y_i^3 &= 0; & \sum_i Y_i &= 0 \end{aligned} \tag{1}$$

- In all these sums, whenever a right-handed field appear, its charge conjugate is considered.
- A Higgsino doublet spoils anomaly cancellation !

Solution to the problem

- Solution: Add a second doublet with opposite hypercharge.
- Anomalies cancel automatically, since the fermions of the second Higgs superfield act as the vector mirrors of the ones of the first one.
- Use the second Higgs doublet to construct masses for the down quarks and leptons.

$$P[\Phi] = h_u QUH_2 + h_d QDH_1 + h_l LEH_1 \quad (2)$$

- Once these two Higgs doublets are introduced, a mass term may be written

$$\delta P[\Phi] = \mu H_1 H_2 \quad (3)$$

- μ is only renormalized by wave functions of H_1 and H_2 .

Higgs Fields

- Two Higgs fields with opposite hypercharge.

(S = 0)

(S = 1/2)

H_1

\tilde{H}_1

(1,2,-1/2)

H_2

\tilde{H}_2

(1,2,1/2)

- It is important to observe that the quantum numbers of H_1 are exactly the same as the ones of the lepton superfield L .
- This means that one can extend the superpotential $P[\Phi]$ to contain terms that replace H_1 by L .

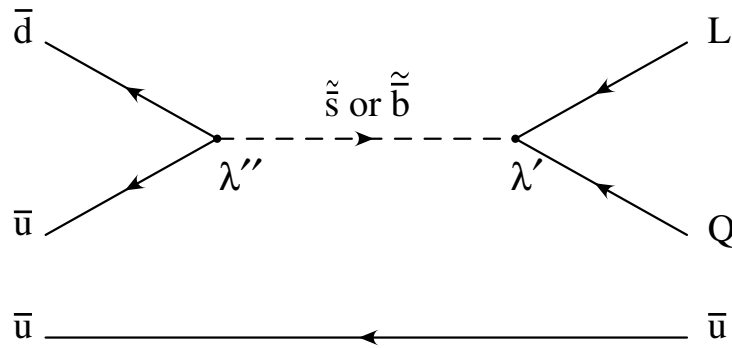
Baryon and Lepton Number Violation

- General superpotential contains, apart from the Yukawa couplings of the Higgs to lepton and quark fields, new couplings:

$$P[\Phi]_{\text{new}} = \lambda' LQD + \lambda LLE + \lambda'' UDD \quad (4)$$

- Assigning every lepton chiral (antichiral) superfield lepton number 1 (-1) and every quark chiral (antichiral) superfield baryon number 1/3 (-1/3) one obtains :
 - Interactions in $P[\Phi]$ conserve baryon and lepton number.
 - Interactions in $P[\Phi]_{\text{new}}$ violate either baryon or lepton number.
- One of the most dangerous consequences of these new interaction is to induce proton decay, unless couplings are very small and/or sfermions are very heavy.

Proton Decay



- Both lepton and baryon number violating couplings involved.
- Proton: Lightest baryon. Lighter fermions: Leptons

R-Parity

- A solution to the proton decay problem is to introduce a discrete symmetry, called R-Parity. In the language of component fields,

$$R_P = (-1)^{3B+2S+L} \quad (5)$$

- All Standard Model particles have $R_P = 1$.
- All supersymmetric partners have $R_P = -1$.
- All interactions with odd number of supersymmetric particles, like the Yukawa couplings induced by $P[\Phi]_{\text{new}}$ are forbidden.
- Supersymmetric particles should be produced in pairs.
- The lightest supersymmetric particle is stable.
- Good dark matter candidate. Missing energy at colliders.

Supersymmetry Breaking

- No supersymmetric particle have been seen: **Supersymmetry is broken in nature**
- Unless a specific mechanism of supersymmetry breaking is known, no information on the spectrum can be obtained.
- **Cancellation of quadratic divergences:**
 - Relies on equality of couplings and not on equality of the masses of particle and superpartners.
- **Soft Supersymmetry Breaking:** Give different masses to SM particles and their superpartners but preserves the structure of couplings of the theory.

Supersymmetry Breaking Parameters

Standard Model quark, lepton and gauge boson masses are protected by chiral and gauge symmetries.

Supersymmetric partners are not protected.

Explanation of absence of supersymmetric particles in ordinary experience/ high-energy physics colliders: Supersymmetric particles can acquire gauge invariant masses, as the one of the SM-Higgs.

Different kind of parameters:

Squark and slepton masses

$$m_{\tilde{q}}^2, m_{\tilde{l}}^2$$

Gaugino (Majorana) masses

$$M_i, \quad i = 1-3$$

Trilinear scalar masses ($\tilde{f}_L^* \tilde{f}_R H_i$)

$$A_f, -\mu^* \quad (\text{This last one comes$$

from the scalar potential derived from the superpotential $|\partial P / \partial A_i|^2$).

They induce mixing between left and right sfermions.

Higgsino Mass μ and associated Higgs Mass Parameters

$$|\mu|^2 + m_{H_i}^2 \quad (\text{The first term may be derived from the superpotential}).$$

Gaugino/Higgsino Mixing

- Just like the gauge boson mixes with the Goldstone modes of the theory after spontaneous breakdown of the gauge symmetry, gauginos mix with the Higgsinos.
- Mixing comes from the interaction $\sqrt{2}gA_i^*T_a\psi_i\lambda^a$, when one takes $A_i \equiv H_i$, and $\lambda^a \equiv \tilde{W}^a, \tilde{B}$, and $\psi_i = \tilde{H}_i$.
- Charged Winos, $\tilde{W}_1 \pm i\tilde{W}_2$, mix with the charged components of the Higgsinos $\tilde{H}_{1,2}$. The mass eigenstates are called **charginos** $\tilde{\chi}^\pm$.
- Neutral Winos and Binos, \tilde{B}, \tilde{W}_3 mix with the neutral components of the Higgsinos. The mass eigenstates are called **neutralinos**, $\tilde{\chi}^0$.
- Charginos form two Dirac massive fields. Neutralinos give four massive Majorana states.

Chargino Mass matrix

Lets take, for instance, the chargino mass matrix in the basis of Winos and Higgsinos, $(\tilde{W}^+, \tilde{H}_2^+)$ and $(\tilde{W}^-, \tilde{H}_1^-)$, with $\tilde{W}^\pm = \tilde{W}^1 \pm i\tilde{W}^2$. The mixing term is proportional to the weak coupling and the Higgs v.e.v.'s

$$M_{\tilde{\chi}^\pm} = \begin{bmatrix} M_2 & g_2 v_2 \\ g_2 v_1 & \mu \end{bmatrix} \quad (6)$$

Here, M_2 is the soft breaking mass term of the Winos and μ is the Higgsino mass parameter.

- The eigenstates are two Dirac, charged fermions (charginos).
- If μ is large, the lightest chargino is a Wino, with mass M_2 , and its interactions to fermion and sfermions are governed by gauge couplings.
- If M_2 is large, the lightest chargino is a Higgsino, with mass μ , and the interactions are governed by Yukawa couplings.

Neutralino Mass Matrix

Similarly, for neutralinos in the basis of Binos, Winos and Higgsinos

$$M_{\tilde{\chi}^0} = \begin{bmatrix} M_1 & 0 & -g_1 v_1 / \sqrt{2} & g_1 v_2 / \sqrt{2} \\ 0 & M_2 & g_2 v_1 / \sqrt{2} & -g_2 v_2 / \sqrt{2} \\ -g_1 v_1 / \sqrt{2} & g_2 v_1 / \sqrt{2} & 0 & -\mu \\ g_1 v_2 / \sqrt{2} & -g_2 v_2 / \sqrt{2} & -\mu & 0 \end{bmatrix} \quad (7)$$

- The eigenstates are four Majorana particles.
- If the theory proceeds from a GUT, there is a relation between M_2 and M_1 , $M_2 \simeq \alpha_2(M_Z) / \alpha_1(M_Z) M_1 \simeq 2M_1$.
- So, if μ is large, the lightest neutralino is a Bino (superpartner of the hypercharge gauge boson) and its interactions are governed by g_1 .
- This tends to be a good dark matter candidate.

Higgs Potential

- After supersymmetry breaking effects are considered, the Higgs potential reads

$$V(H_1, H_2) = m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 + m_3^2 (H_1^T i\tau_2 H_2 + h.c.) + \frac{\lambda_1}{2} (H_1^\dagger H_1)^2 + \frac{\lambda_2}{2} (H_2^\dagger H_2)^2 + \lambda_3 (H_1^\dagger H_1) (H_2^\dagger H_2) + \lambda_4 \left| (H_1^\dagger H_2) \right|^2$$

where

$$\lambda_1 = \lambda_2 = \frac{g_1^2 + g_2^2}{4}, \quad \lambda_3 = \frac{g_2^2 - g_1^2}{4}, \quad \lambda_4 = -\frac{g_2^2}{2} \quad (8)$$

- This effective potential is valid at the scale of the SUSY particle masses.
- The value of the effective potential at low energies may be obtained by evolving the quartic couplings with their renormalization group equations.

Evolution of Parameters

- In general, the parameters that one measures at low energies (large distances) are not the fundamental ones, but they are modified by quantum corrections.
- For instance, if you put a charge into the vacuum state, it will polarize the vacuum by inducing the production of virtual particles and antiparticles, which “screen” the original charge.
- This also happens with other couplings and also with mass parameters. There are equations, called renormalization group equations that allow to relate the fundamental parameters to the ones at low energies.

Higgs Boson Mass

- The RG evolution of λ_2 is given by

$$\frac{d\lambda_2}{dt} \simeq -\frac{3}{8\pi^2} [\lambda_2^2 + \lambda_2 h_t^2 - h_t^4] \quad (9)$$

with $t = \log(M_{SUSY}^2/Q^2)$.

- For large values of $\tan \beta = v_2/v_1$, the Higgs H_2 is the only one associated with electroweak symmetry breaking.
- The Higgs boson mass is approximately given by $m_h^2 = 2\lambda_2 v^2$

$$m_h^2 \simeq M_Z^2 + \frac{3m_t^4}{4\pi^2 v^2} \left[\log \left(\frac{M_{SUSY}^2}{m_t^2} \right) + \frac{A_t^2}{M_{SUSY}^2} \left(1 - \frac{A_t^2}{12M_{SUSY}^2} \right) \right] \quad (10)$$

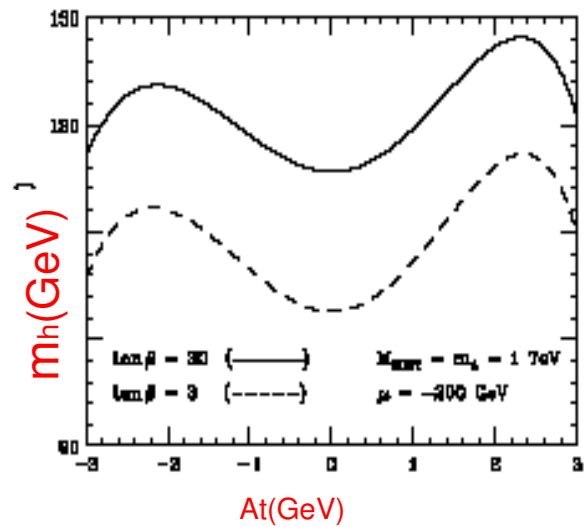
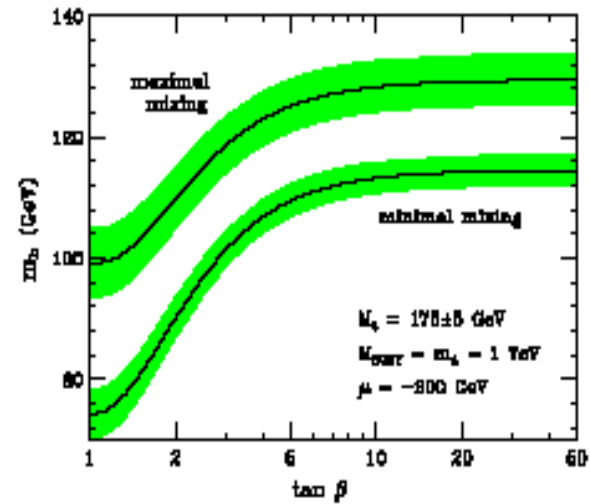
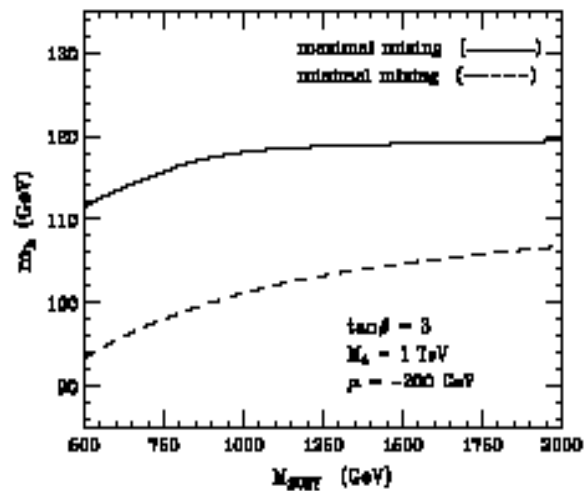
- The first term comes from the SUSY contribution. The logarithmic term comes from the RG evolution, while the A_t dependence comes from threshold effects at M_{SUSY} .

Stop Mass Matrix

- The stop, and other squarks, acquire masses that are controlled by the supersymmetry breaking parameters.
- Once the Higgs acquires a v.e.v., the mass matrix is

$$M_{\tilde{t}}^2 = \begin{bmatrix} m_Q^2 + m_t^2 & m_t(A_t - \mu^* / \tan \beta) \\ m_t(A_t^* - \mu / \tan \beta) & m_U^2 + m_t^2 \end{bmatrix} \quad (11)$$

- In general, the existence of A_t and μ denote couplings of the stops to the Higgs bosons, that induce finite corrections to the quartic couplings.



- m_t^4 enhancement
 - logarithmic sensitivity to $m_{\tilde{t}_i}$
 - depend. on \tilde{t} -mixing X_t
- \Rightarrow max. value $X_t \sim \sqrt{6}M_S$

Carena, Haber, Hollik, Heinemeyer, Weiglein, C.W. '00 .O.
 Heinemeyer, Hollik, Weiglein'02
 Degraess, Slavich, Zwirner '02

$$M_{SUSY} \equiv M_Q = M_U = M_D \quad \text{if } M_{SUSY} \gg m_t \rightarrow M_S^2 \simeq M_{SUSY}^2$$

- at 2 loops $\rightarrow M_{\tilde{g}}$ dependence

Higgs Spectrum

- The two Higgs doublets carry eight real scalar degrees of freedom.
- Three of them are the charged and CP-odd Goldstone bosons that are absorbed in the longitudinal components of the W and the Z .
- Five Higgs bosons remain: Two CP-even, one CP-odd, neutral bosons, and a charged Higgs boson (two degrees of freedom).
- Generically, the electroweak breaking sector (Goldstones and real Higgs) is contained in the combination of doublets

$$\Phi = \cos \beta H_1 + \sin \beta i\tau_2 H_2^*, \quad (12)$$

while the orthogonal combination contains the other Higgs bosons.
Their masses are:

$$m_H^2 \simeq m_A^2, \quad m_{H^\pm}^2 \simeq m_A^2 + M_W^2 \quad (13)$$

These relations are preserved, in a good approximation, after loop-effects.

Counting degrees of Freedom

- The charginos are two Dirac particles, with eight degrees of freedom, and are an admixture of the superpartners of the charged gauge bosons and Higgs bosons.
- The boson sector has the W^\pm , that has six degrees of freedom, plus the charged Higgs, with two degrees of freedom. Observe that before electroweak symmetry breaking there is no mixing and the numbers are four and four, respectively.
- Neutralinos are four Majorana particles, and have eight degrees of freedom.
- There are five in the neutral gauge bosons (photon and Z) plus three in the neutral Higgs bosons. Again, before electroweak symmetry breaking, the numbers were four and four.

Minimal Supergravity Model

All scalars acquire a common mass m_0^2 at the Grand Unification scale

All gauginos acquire a common mass $M_{1/2}$ at the GUT scale

Masses evolve differently under R.G.E. At low energies,

Squark Masses: $m_{\tilde{Q}}^2 \simeq m_0^2 + 6 M_{1/2}^2$

Left-Slepton Masses $m_{\tilde{L}}^2 \simeq m_0^2 + 0.5 M_{1/2}^2$

Right-Slepton Masses $m_{\tilde{E}}^2 \simeq m_0^2 + 0.15 M_{1/2}^2$

Wino Mass $M_2 = 0.8 M_{1/2}$.

Gluino Mass $M_3 = \frac{\alpha_3}{\alpha_2} M_2$

Bino Mass $M_1 = \frac{\alpha_1}{\alpha_2} M_2$

Lightest SUSY particle tends to be a Bino.

Electroweak Symmetry Breaking

- The above relations apply to most squarks and leptons, but not to the Higgs particles and the third generation squarks.
- The renormalization group equations of these mass parameters include negative corrections proportional to the square of the large top Yukawa coupling.
- In particular, the H_2 Higgs mass parameter m_2^2 , is driven to negative values due to the influence of the top quark Yukawa coupling.
- Electroweak symmetry breaking is induced by the large top mass !
- Also the superpartners of the top quark tend to be lighter than the other squarks. This effect is more pronounced if $M_{1/2}$ is small.

If SUSY exists, many of its most important motivations demand some SUSY particles at the TeV range or below

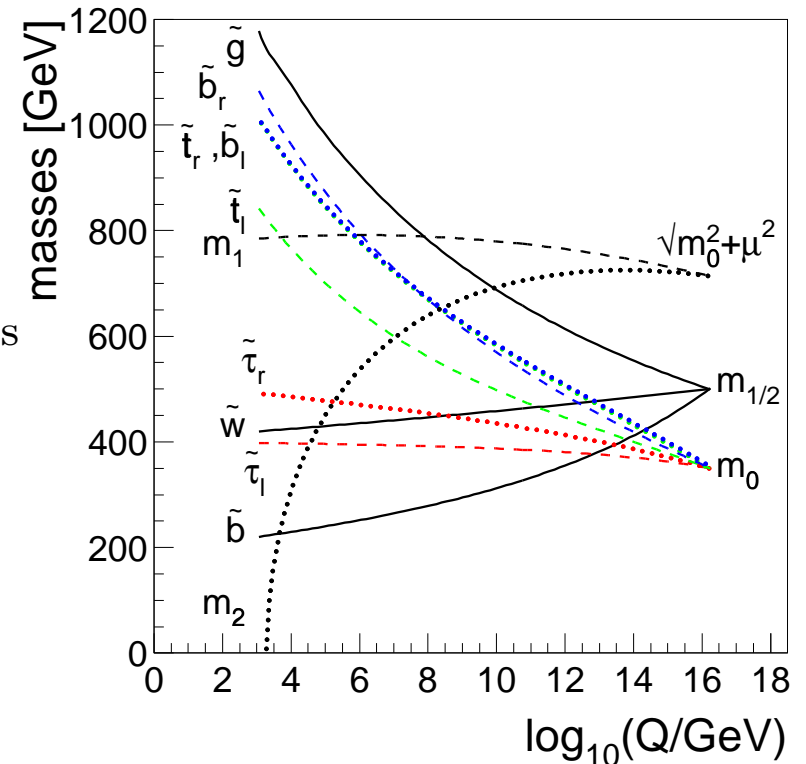
★ Solve hierarchy/naturalness problem by having $\Delta m^2 \simeq \mathcal{O}(v^2)$

SUSY breaking scale must be at or below 1 TeV
if SUSY is associated with EWSB scale !

★ EWSB is radiatively generated

In the evolution of masses from high energy scales
→ a negative Higgs mass parameter is induced
via radiative corrections

⇒ *important top quark effects!*

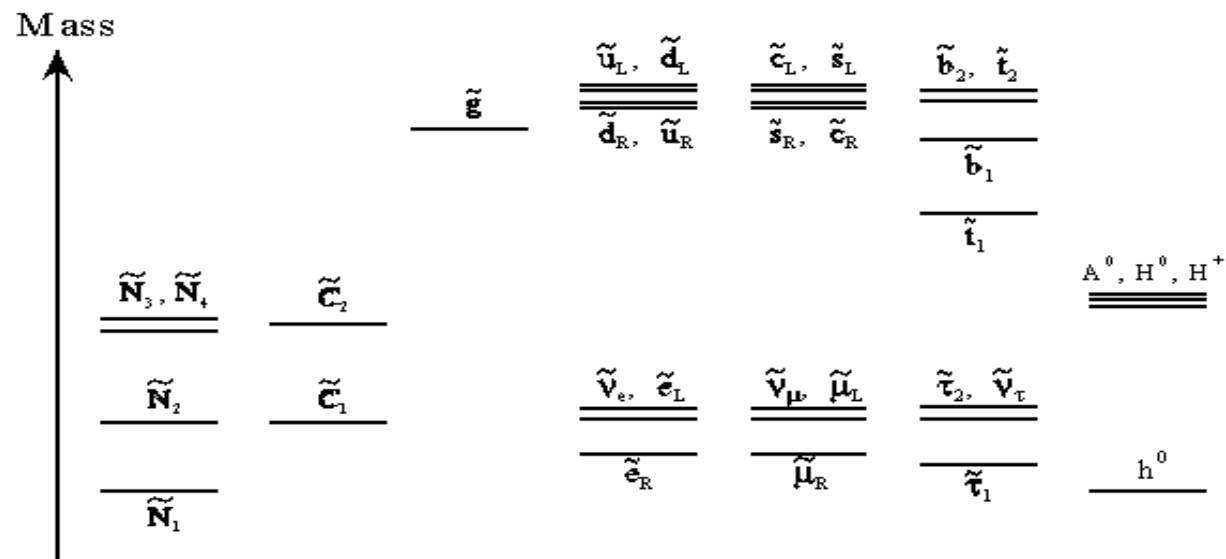


SUSY Spectrum from Universal Boundary Conditions at M_{GUT}

Coloured, strongly interacting particles tend to be heavier than weakly interacting particles.

Lightest Supersymmetric Particle is lightest Neutralino.

Squarks and sleptons of first two generations are degenerate. Small splitting in the slepton sector. Third generation squarks are Typically lighter.



Comments

- The previously presented spectrum depends strongly on the condition of equality of sfermion and gaugino masses at the GUT scale.
- Setting, for instance, different masses for particles of different quantum numbers at the GUT scale could lead to a very different spectrum.
- In general, very little is known about the supersymmetry breaking parameters and one should NOT make conclusions about the Tevatron and/or LHC reach for SUSY based on strong assumptions about them.
- In particular, although it is clear that the LHC has a larger reach, the Tevatron one is not at all negligible, and one should be open to the possibility of a SUSY discovery before the start of the LHC !

Feynman Rules for Supersymmetric Theories



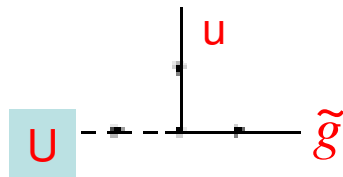
$$-ig_3 T^a \gamma^\mu$$

Start with SM couplings



$$-ig_3 T^a (p + k)^\mu$$

Change fermion by scalars
and gammas by momentum



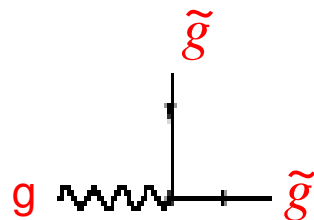
$$-ig_3 T^a (c_U P_L + s_U P_R)$$

Change one fermion by scalar
and gluons by gluinos and
gammas by constants (Yukawa
Couplings). Extra factors are
mixing angles that project mass
eigenstates into gauge
eigenstates.



$$-ig_3 T^a (c_D P_L + s_D P_R)$$

Gluons and Gluinos



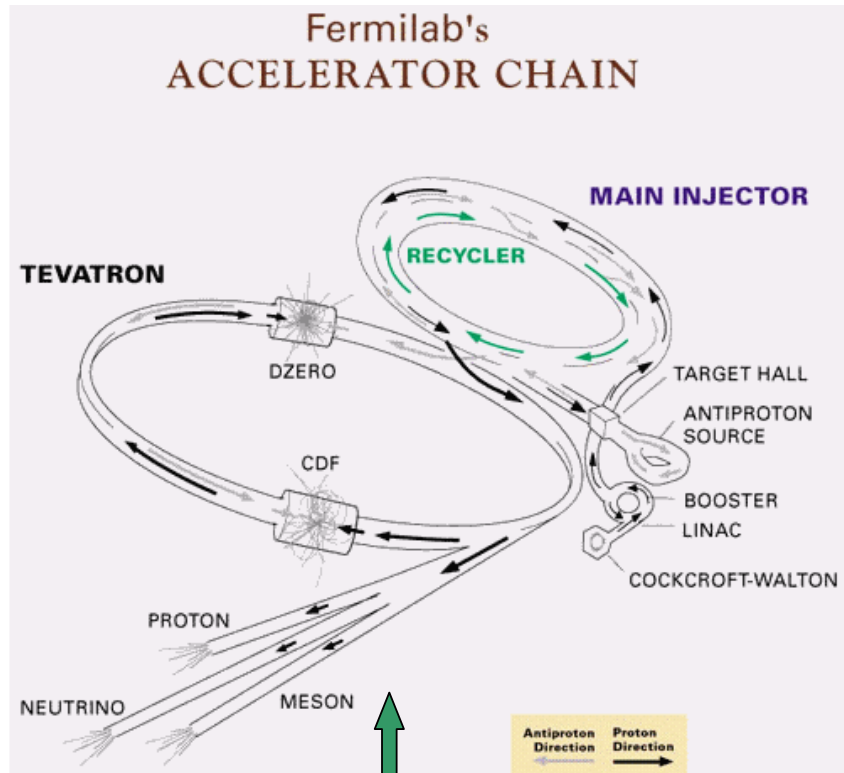
$$- g_3 f_{abc} \gamma^\mu$$



$$- g_3 f_{abc} \left[g^{\mu\nu} (k_1 - k_2)^\lambda + g^{\nu\lambda} (k_2 - k_3)^\mu + g^{\mu\lambda} (k_3 - k_1)^\nu \right]$$

Gluinos are strongly interacting particles and, unless very heavy, are one of the most copiously produced particles at hadron colliders

Searching for New Physics

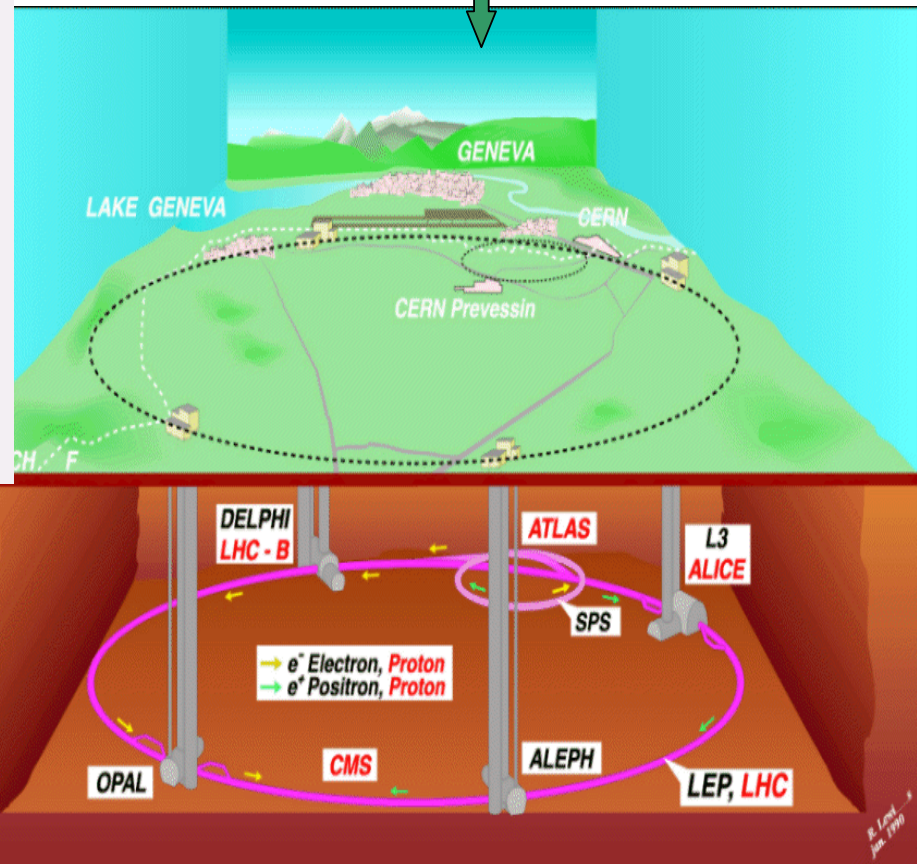


The Tevatron at Fermilab

$p\bar{p}$ at $\sqrt{s} = 2 \text{ TeV}$ (2001 - 2009)

Large Hadron Collider (LHC) at CERN

pp at $\sqrt{s} = 14 \text{ TeV}$ (2007/8 - 2015/20)



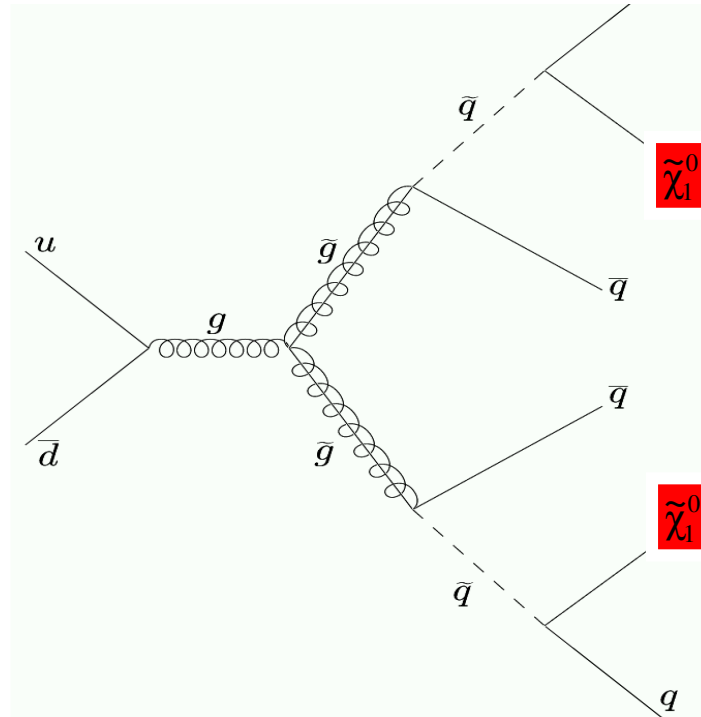
Supersymmetry at colliders

Gluino production and decay: Missing Energy Signature

*Supersymmetric
Particles tend to
be heavier if they
carry color charges.*

*Particles with large
Yukawas tend to be
lighter.*

*Charge-less particles
tend to be the
lightest ones.*

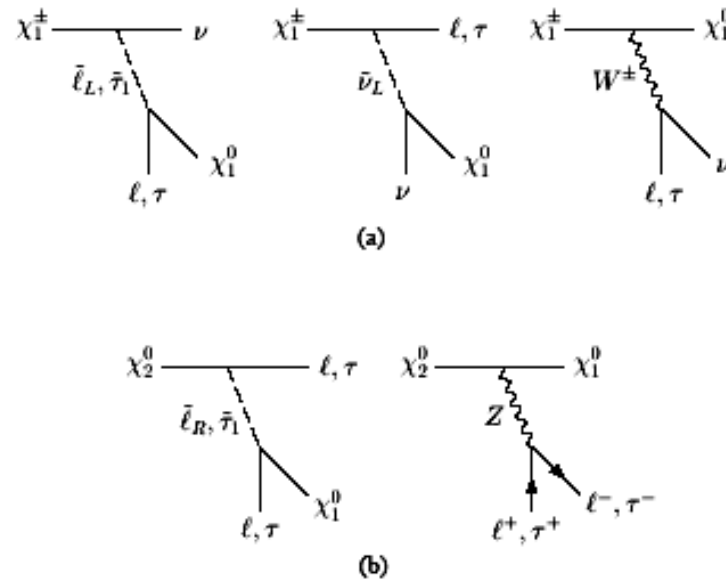
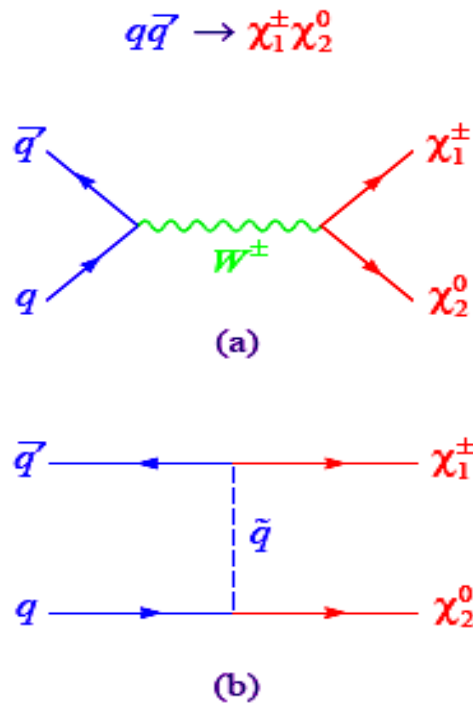


➤ **Lightest supersymmetric particle = Excellent
Cold dark matter candidate.**

Trilepton Signatures at the Tevatron

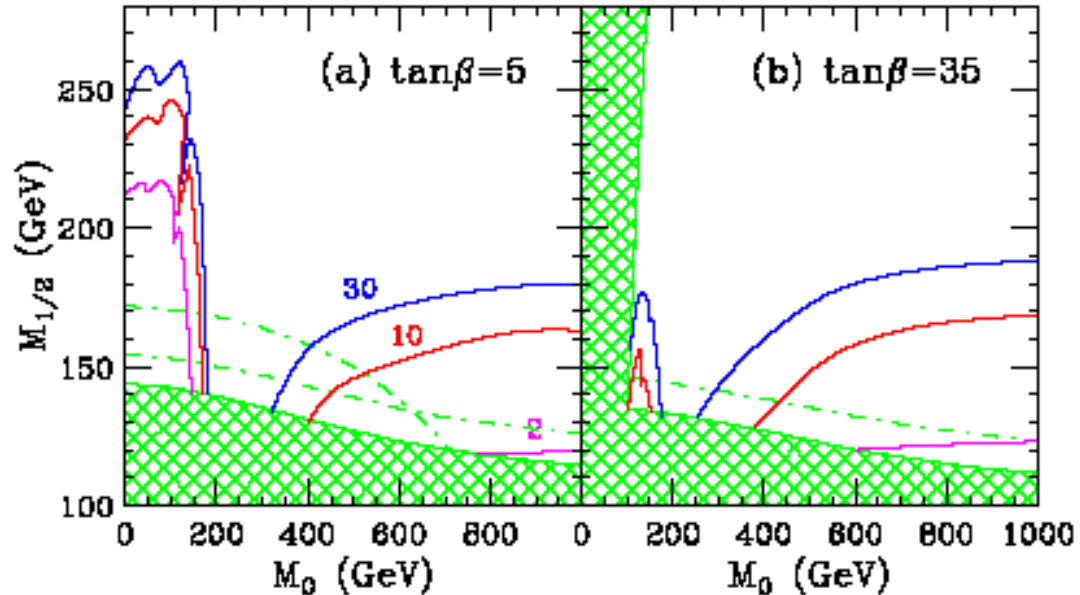
Trileptons are associated production of charginos and neutralinos, with subsequent decays into leptons. The final signal is three leptons and plenty of missing energy (neutralinos and neutrinos)

Main background: W and Z production.



Trilepton Tevatron reach in MSUGRA

Best parameter region is associated with relatively light charginos (lighter than about 200 GeV), and relatively light squarks. Results depend sensitively on the squark masses, due to interference of different chargino-neutralino production diagrams.



Large Hadron Collider (LHC)

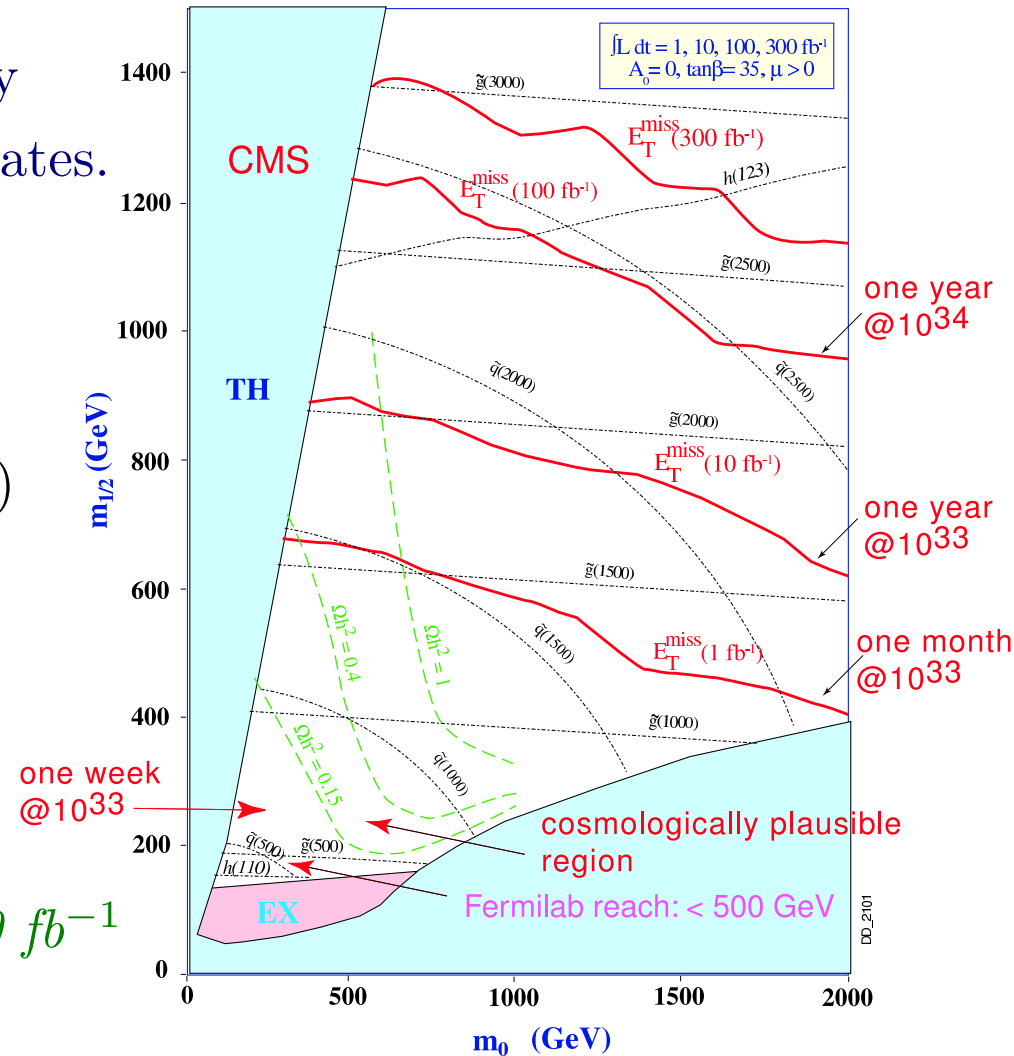
CERN, Geneva, Switzerland

SUSY

LHC: SUSY particles, especially strongly interacting ones, are produced at large rates.

- most likely types of signatures:
 - ‘mSUGRA’ type –
high E_T jets and \cancel{E}_T (maybe leptons)

reach: $M_{\tilde{q}}$ and $M_{\tilde{g}}$ up to $\sim 2 \text{ TeV}$ with 10 fb^{-1}



If low-energy SUSY is there, we expect to see some of its signature(s) by the end of this decade.

Alternative SUSY Breaking scenario: Gauge Mediation

Supersymmetry Breaking is transmitted via gauge interactions

Particle Masses depend on the strength of their gauge interactions.

Spectrum of supersymmetric particles very similar to the case of the Minimal Supergravity Model for large values of $M_{1/2}$:

Sparticle Masses

$$\frac{M_i}{M_j} = \frac{\alpha_i}{\alpha_j}$$

$$\frac{m_{\tilde{q}}}{m_{\tilde{l}}} \simeq \frac{\alpha_3}{\alpha_i} \quad (m_{\tilde{f}} \simeq \frac{\alpha}{4\pi} \frac{F}{M})$$

Lightest SM–Sparticle tends to be a Bino or a Higgsino

Gravitino

- When standard symmetries are broken spontaneously, a massless boson appears for every broken generator.
- If the symmetry is local, these bosons are absorbed into the longitudinal components of the gauge bosons, which become massive.
- The same is true in supersymmetry. But now, a massless fermion appears, called the Goldstino.
- In the case of local supersymmetry, this Goldstino is absorbed into the Gravitino, which acquires mass $m_{\tilde{G}} = F/M_{Pl}$, with F the order parameter of SUSY breaking.
- The coupling of the Goldstino (gravitino) to matter is proportional to $1/\sqrt{F} = 1/\sqrt{m_{\tilde{G}}M_{Pl}}$, and couples particles with their superpartners.
- Masses of supersymmetric particles is of order F/M , where M is the scale at which SUSY is transmitted.

Gauge-Mediated, Low-energy SUSY Breaking Scenarios

- Special feature \longrightarrow LSP: light (gravitino) Goldstino:

$$m_{\tilde{G}} \sim \frac{F}{M_{Pl}} \simeq 10^{-6} - 10^{-9} \text{GeV}$$

If R-parity conserved, heavy particles cascade to lighter ones and
NLSP \longrightarrow SM partner + \tilde{G}

- Signatures: The NLSP (Standard SUSY particle) decays

decay length $L \sim 10^{-2} \text{cm} \left(\frac{m_{\tilde{G}}}{10^{-9} \text{GeV}} \right)^2 \times \left(\frac{100 \text{GeV}}{M_{\text{NLSP}}} \right)^5$

★ NLSP can have prompt decays:

Signature of SUSY pair: 2 hard photons, (H's, Z's) + \cancel{E}_T from \tilde{G}

★ macroscopic decay length but within the detector:

displaced photons; high ionizing track with a kink to a minimum ionizing track
(smoking gun of low energy SUSY)

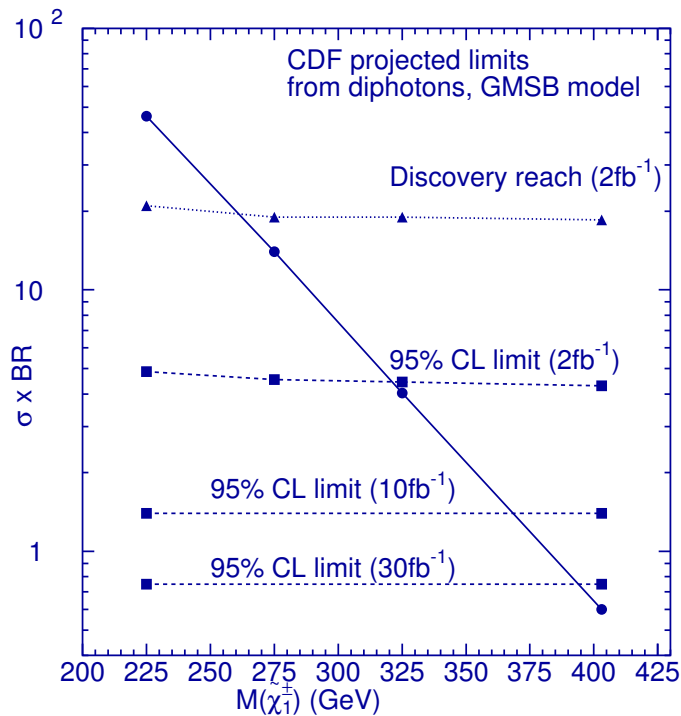
★ decay well outside the detector: \cancel{E}_T like SUGRA

Gauge-Mediated Tevatron Reach

■ Bino-like NLSP: $\tilde{\chi}_1^0 \rightarrow \gamma \tilde{G}$

Signal: $\gamma\gamma X \cancel{E}_T$

$X = \ell$'s and/or jets

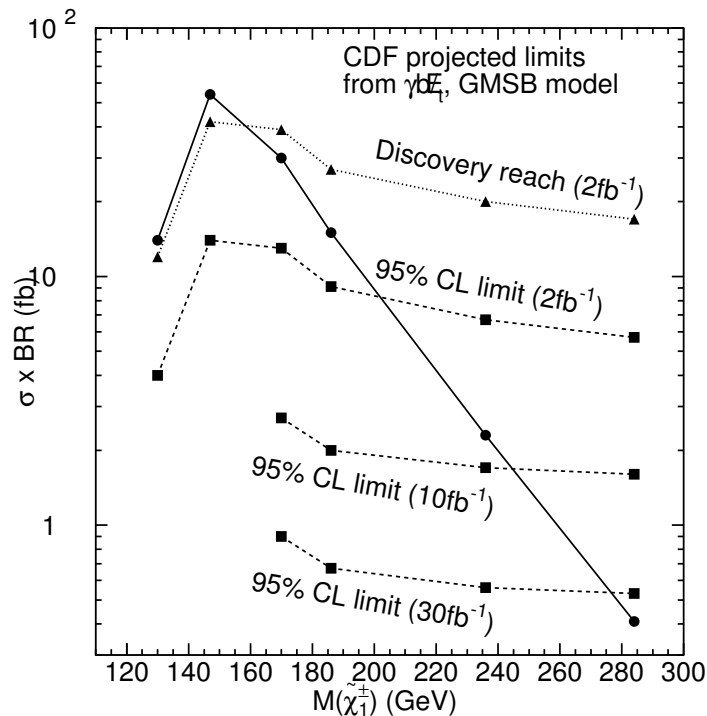


$M_{\tilde{\chi}^\pm} \sim 325 \text{ GeV}$ (exclusion) &
 $\sim 260 \text{ GeV}$ (discovery)

■ Higgsino-like NLSP: $\tilde{\chi}_1^0 \rightarrow (h, Z, \gamma) \tilde{G}$

Signal: $\gamma b \cancel{E}_T X$

diboson signatures ($Z \rightarrow \ell\ell/\text{jj}$; $h \rightarrow b\bar{b}$) \cancel{E}_T



$M_{\tilde{\chi}_1^\pm}$ sensitivity $\sim 200 \text{ GeV}$ for 2 fb^{-1}

Light Stops: Motivation

- In low energy supersymmetry models, light stops are induced as a consequence of large mixing or large negative radiative effects.
- They are required for the realization of the mechanism of electroweak baryogenesis in the MSSM
- Signatures of a light stop at the Tevatron collider depend strongly on the chargino and neutralino spectrum as well as on the nature of supersymmetry breaking

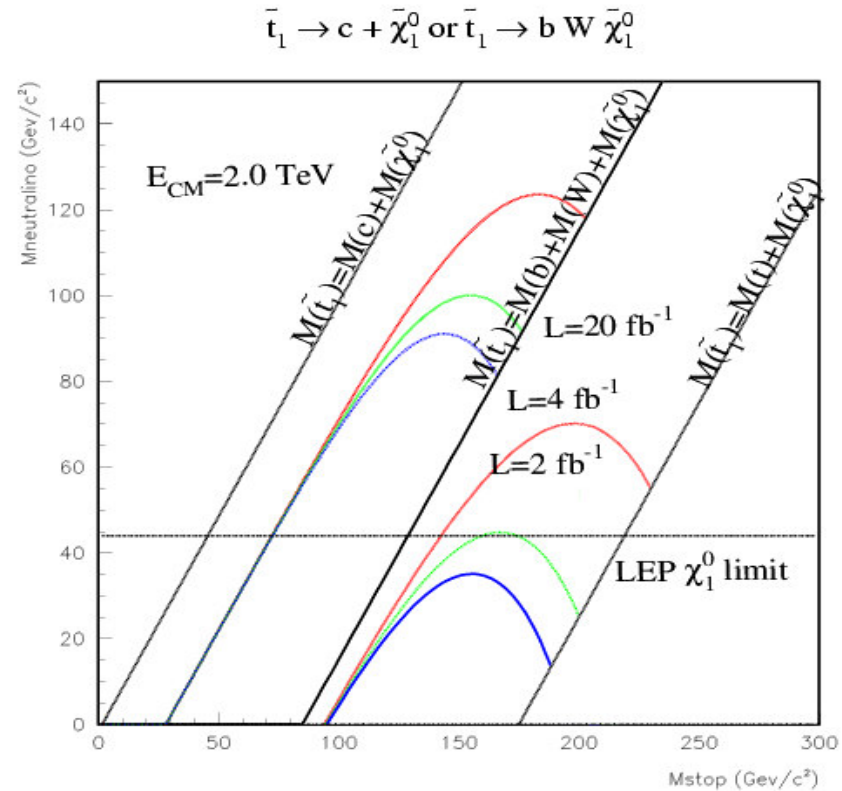
Tevatron Stop Reach when two body decay channel is dominant

Main signature:

2 or more jets plus
missing energy

2 or more Jets with $E_T > 15$ GeV

Missing $E_T > 35$ GeV

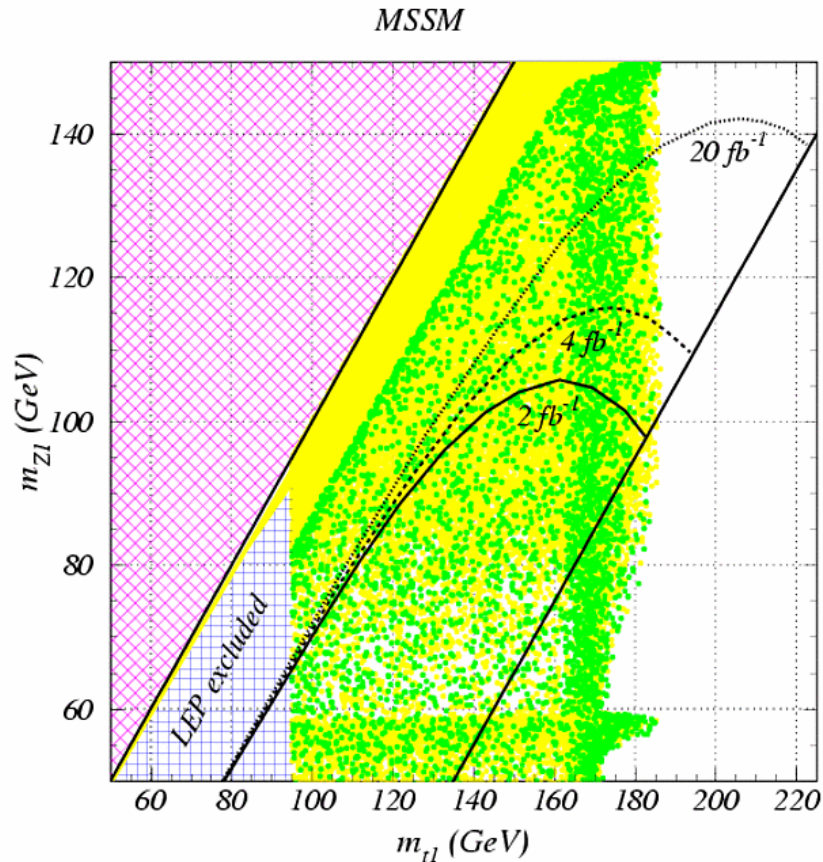


Demina, Lykken, Matchev, Nomerotsky '99

Tevatron Run II reach for stops probes

Dark Matter and Baryogenesis at the Electroweak scale!

Dots show scan over SUSY space with neutralino relic density compatible with WMAP observations $0.095 \leq \Omega_{CDM} h^2 \leq 0.129$ and with electroweak Baryogenesis



Balazs, Carena & C.W.

Lines show reach at the Tevatron for different total luminosities for dominant decay mode $\tilde{t}_1 \rightarrow c \tilde{\chi}$.

If stop-neutralino mass difference is below 30 GeV:

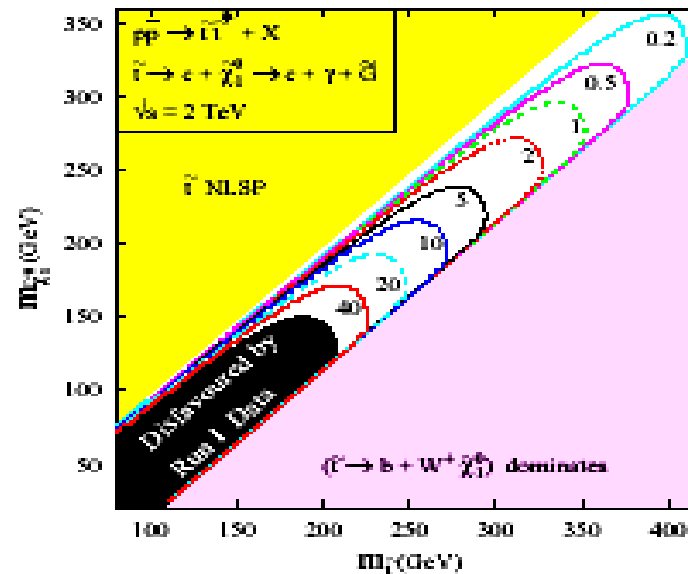
- trigger on \cancel{E}_T crucial
- co-annihilation region difficult at Tevatron or any hadron collider

A definite test of this scenario at the LC

Tevatron stop searches in low-energy SUSY breaking models

Carena, Choudhury, Diaz,
Logan and C.W. '02

Extra photon and large
missing energy helpful
in stop detection



Cross sections for stop pair production
in fb , with $\tilde{t} \rightarrow c\gamma\tilde{G}$ and Signal/selection
 $jj\gamma\gamma\cancel{E}_T$

$\int \mathcal{L}$	$\sigma_S \ 5\sigma$	Max. $m_{\tilde{t}}$ (2 body)
2 fb^{-1}	6 fb	290 GeV
4 fb^{-1}	3.5 fb	315 GeV

Unification of Couplings

- The value of gauge couplings evolve with scale according to the corresponding RG equations:

$$\frac{1}{\alpha_i(Q)} = \frac{b_i}{2\pi} \ln \left(\frac{Q}{M_Z} \right) + \frac{1}{\alpha_i(M_Z)} \quad (14)$$

- Unification of gauge couplings would occur if there is a given scale at which couplings converge.

$$\frac{1}{\alpha_3(M_Z)} = \frac{b_3 - b_2}{b_1 - b_2} \frac{1}{\alpha_1(M_Z)} + \frac{b_3 - b_1}{b_2 - b_1} \frac{1}{\alpha_2(M_Z)} \quad (15)$$

- This leads to a relation between $\alpha_3(M_Z)$ and $\sin^2 \theta_W(M_Z) = \alpha_1^{SM} / (\alpha_1^{SM} + \alpha_2^{SM})$.

Rules to compute the beta-functions

- The one-loop beta-functions for the $U(1)$ and $SU(N)$ gauge couplings are given by,

$$\begin{aligned}\frac{5}{3}b_1 &= -\frac{1}{6}\sum_f y_f^2 - \frac{1}{12}\sum_s y_s^2 \\ b_N &= \frac{11}{3}N - \frac{n_f}{3} - \frac{n_S}{6} - \frac{2N}{3}n_A\end{aligned}\tag{16}$$

- In the above, $y_{f,s}$ are the hypercharges of the charged chiral fermion and scalar fields, $n_{f,s}$ are the number of fermions and scalars in the fundamental representation of $SU(N)$, n_A are fermions in the adjoint representation and the factor $5/3$ is just a normalization factor, so that over one generation

$$Tr [T^3 T^3] = \frac{3}{5} Tr \left[\left(\frac{y_f}{2} \right)^2 \right]\tag{17}$$

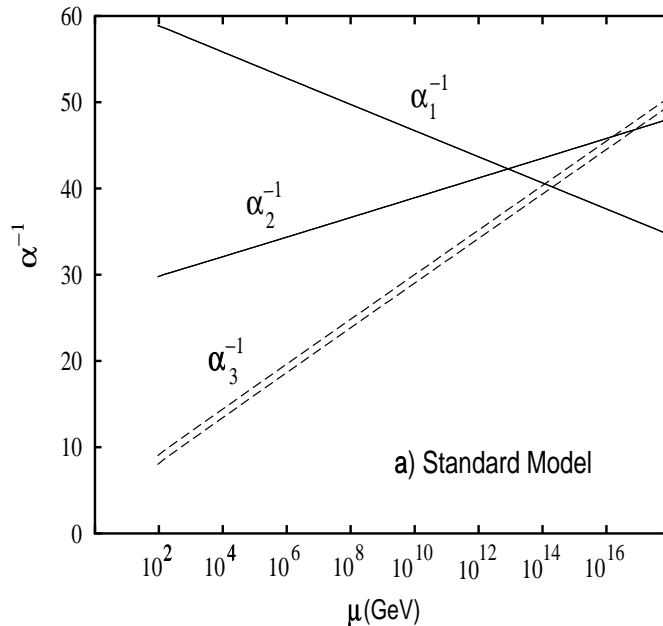
- The coupling g_1 is not asymptotically free, but becomes strong at scales far above M_{Pl} , where the effective theory description breaks down anyway.
- A full generation contributes to the same amount to all β -functions. It does not affect the unification conditions. ($\beta_{\text{gen}}^{SM} = 4/3$; $\beta_{\text{gen}}^{\text{SUSY}} = 2$.)
- Only incomplete $SU(5)$ representations affect one-loop unification.

$$\begin{aligned}
 b_1^{SM} &= -41/10, & b_2^{SM} &= 19/6, & b_3^{SM} &= 7. \\
 b_1^{\text{SUSY}} &= -33/5, & b_2^{\text{SUSY}} &= -1, & b_3^{\text{SUSY}} &= 3.
 \end{aligned} \tag{18}$$

$$\begin{aligned}
 M_G &= M_Z \exp \left[\left(\frac{1}{\alpha_1(M_Z)} - \frac{1}{\alpha_2(M_Z)} \right) \frac{2\pi}{b_2 - b_1} \right] \\
 \frac{1}{\alpha_3(M_Z)} &= \frac{b_3 - b_1}{b_2 - b_1} \frac{1}{\alpha_2(M_Z)} + \frac{b_3 - b_2}{b_1 - b_2} \frac{1}{\alpha_1(M_Z)}
 \end{aligned} \tag{19}$$

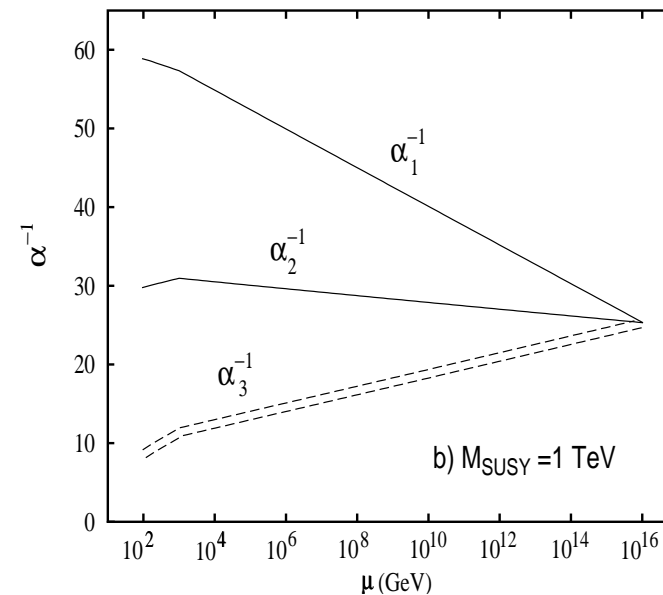
SM:

Couplings tend to converge at high energies, but unification is quantitatively ruled out.



MSSM:

Unification at $\alpha_{GUT} \simeq 0.04$ and $M_{GUT} \simeq 10^{16}$ GeV.



Experimentally, $\alpha_3(M_Z) \simeq 0.118 \pm 0.004$ Bardeen, Carena, Pokorski & C.W.
in the MSSM: $\alpha_3(M_Z) = 0.127 - 4(\sin^2 \theta_W - 0.2315) \pm 0.008$

Remarkable agreement between Theory and Experiment!!

Conclusions

- Supersymmetry is a symmetry that relates boson to fermion degrees of freedom. It provides the basis for an extension of the SM description of particle interactions.
- Fundamental property: No new couplings. Masses of supersymmetric particles depend on supersymmetry breaking scheme.
- If R-Parity is imposed and the gravitino is heavier than the lightest SM partner, then the lightest supersymmetric particle is a good dark matter candidate.
- Electroweak symmetry breaking is induced radiatively in a natural way, provided sparticle masses are of order 1 TeV. Unification of couplings is achieved.
- Signature at colliders: Missing Energy, provided by the LSP.
- If low energy susy breaking (gauge mediation) gravitino is LSP and reach of Tevatron is enhanced.

Linear Collider (LC)

Location Unknown

Supersymmetry at a LC

(a) Measurements of SUSY particles masses

⇒ sleptons, charginos, neutralinos

with an accuracy of 1% or less

If any visible SUSY particle produced,

→ $\delta M_{\tilde{\chi}_1^0} \sim 1\% \Rightarrow$ important for LHC meas.

(b) Measurement of SUSY parameters

- $\tilde{\chi}_i^\pm, \tilde{\chi}_i^0$ production & decay
 - param. of mixing mass matrix to 1%
 - determine composition in terms of SUSY partners of γ, Z, W, H
- slepton and squark mixing angles
 - from cross sections with polarized beams

(c) Spin of SUSY particles:

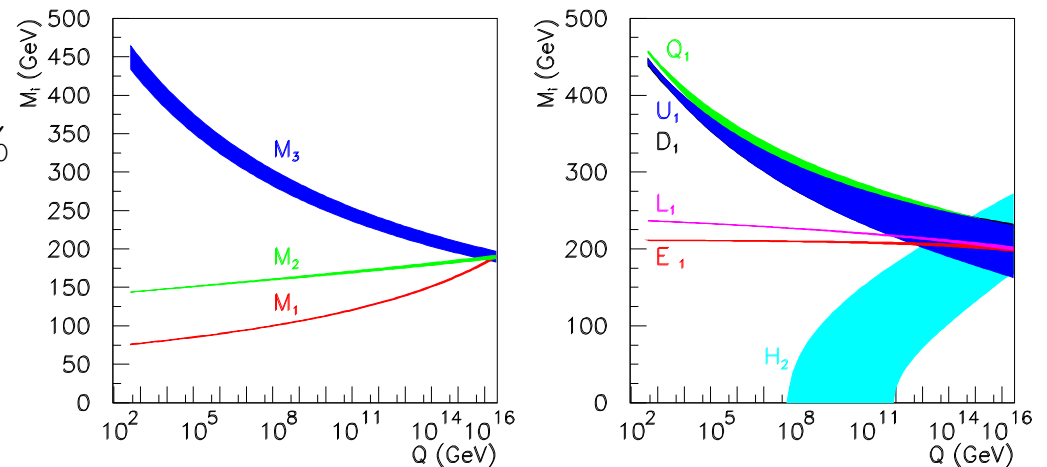
Simplicity of production reactions allows spin determination from angular distributions

Precise SUSY measurements at LC

+ LHC input on gluinos/squarks

⇒ allow for precise extrapolation of SUSY parameters at high energies

Test type of SUSY theory at high energies.



TeV scale Physics can provide our first glimpse of the Planck scale regime!!

Linear Collider and the Cosmos

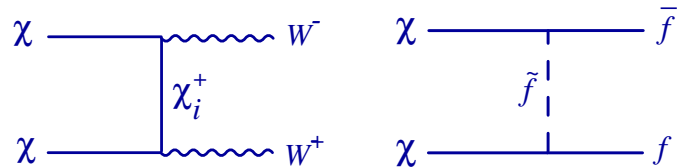
- Weak-interacting particles with weak-scale masses naturally provide Ω_{DM} .

⇒ A coincidence or DM provides fundamental motivation for new particles at EW scale.

- ★ Understanding what DM is made of demands Collider & Astrophysical/Cosmological input.

- If the LSP is found to be a stable neutralino
→ accurate meas. of $\tilde{\chi}_1^0$ mass & composition

⇒ Comput. of $\tilde{\chi}_1^0 \tilde{\chi}_1^0$ annih. cross section



⇒ determined thermal relic density
assuming standard evolution of the universe

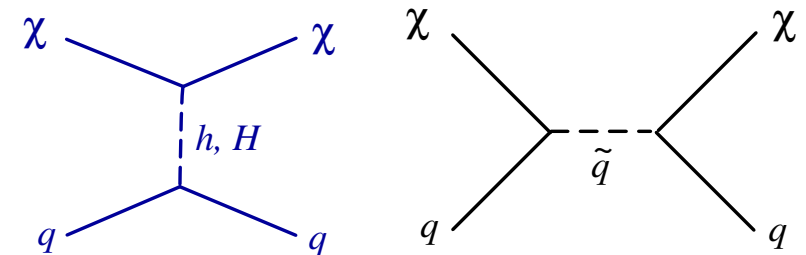
- comparing this result with Ω_{DM} from Astrophysical/Cosmological input

⇒ new insights into history of our universe

- Dark Matter Detection:

- Direct: depends on $\tilde{\chi}_1^0 N$ scattering

→ input from both collider and conventional DM experiments



- Indirect: through annih. decay products
($\tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow \gamma$'s in galactic center, e^+ 's in halo, anti-protons, ν 's in centers of Earth & Sun)

⇒ $\tilde{\chi}_1^0 N$ scattering not necessarily in one-to-one correspondence with DM detection rates

⇒ LC will provide important info about DM halo densities and velocity distributions.